

THERE NEED BE NO SPLIT SCOPE

Peter Alrenga (Boston University) and Christopher Kennedy (University of Chicago)

Like their counterparts in Dutch and German, negative DPs in English participate in “split scope” readings, in which another operator scopes between their negative and existential components. For example, (1) is ambiguous between the reading in (1a), in which negation scopes below the modal, and the split scope reading in (1b), in which negation scopes above the modal, but the rest of the nominal (including existential force) is below it, apparently “splitting” the scope of the DP.

- (1) The company is required to fire no employees.
- It is necessary that the company not fire any employees.
 - It is not necessary that the company fire any employees.

Previous analyses of split scope syntactically decompose negative DPs into a positive indefinite and a covert negation (e.g., Penka 2011), or else invoke special kinds of quantification (Geurts 1996, de Swart 2000, Abels and Martí 2011). In this paper, we argue that split scope need not be explained by appeal to special mechanisms for scope assignment, but is rather a standard case of DEGREE PHRASE MOVEMENT, an idea first suggested in Heim 2001 but not worked out there. We show that English *no* has a denotation as a generalized quantifier over degrees, and that this fact, when coupled with independent assumptions about the presence of a degree argument position in nominals, derives split scope readings for English negative DPs. An important advantage of this analysis is that it automatically derives constraints on split scope readings in English that are identical to those governing the scope of other degree quantifiers.

We begin by observing that in addition to its occurrence in the nominal projection, English *no* also appears as a differential term in comparatives, as illustrated in (2).

- (2) Frank’s income is no higher than \$74,999.

It is well-established that comparatives are type $\langle d, et \rangle$, with the degree argument representing the difference between the degrees to which the target and standard of comparison possess the property expressed by the gradable predicate. Without worrying about the details of the internal compositional semantics (which do not concern us here), this means that *higher than \$74,999* denotes the relation in (3).

- (3) $\llbracket \text{higher than } \$74,999 \rrbracket = \lambda d \lambda x . \mathbf{high}(x) - \$74,999 = d$

In the absence of an overt differential, the degree argument is existentially closed, and the comparative denotes a property that is true of an object iff there is a positive difference between its value on the relevant scale and the value of the standard. The differential *no*, on the other hand, triggers the entailment that there is no such difference. We account for this by treating *no* as a generalized degree quantifier:

- (4) $\llbracket \text{no} \rrbracket = \lambda P_{dt} . \{d : P(d)\} = \emptyset$

Since *no* is type $\langle dt, t \rangle$, it must take scope. The LF of (1) is therefore as shown in (5a), which is assigned the truth conditions in (5b): true just in case there is no positive difference between Frank’s income and \$74,999. This will be the case whenever Frank’s income is \$74,999 or less, which is just what we want.

- (5) a. $[\text{no}_1 [\text{Frank’s income is } [t_1 \text{ higher than } \$74,999]]]$
b. $\{d : \mathbf{high}(\mathbf{f-inc}) - \$74,999 = d\} = \emptyset$

This analysis predicts scopal interactions between *no* and other operators, a prediction that is confirmed by the fact that (6) is ambiguous. (A similar ambiguity is seen with existential modals like *allowed to*.)

- (6) Frank’s income needs to be no higher than \$74,999.
- It is necessary that Frank’s income not exceed \$74,999.
 - It is not necessary that Frank’s income exceed \$74,999.

The readings in (6a) and (6b) derive from LFs in which *no* scopes below and above the modal, respectively, and which receive the truth conditions in (7a) and (7b). (‘@’ stands for the actual world.)

- (7) a. $\forall w [ACC_{@}(w) \rightarrow \{d : \mathbf{high}_w(\mathbf{f-inc}) - \$74,999 = d\} = \emptyset]$
b. $\{d : \forall w [ACC_{@}(w) \rightarrow \mathbf{high}_w(\mathbf{f-inc}) - \$74,999 = d]\} = \emptyset$

The truth conditions in (7a) correspond to the stronger reading in (6a), and state that in every accessible world, there is no positive difference between Frank’s income and \$74,999. The truth conditions in (7b) correspond to the weaker reading in (6b), and state that there is no degree *d* that constitutes a positive difference between Frank’s income and \$74,999 in every accessible world, from which it follows that there are at least some worlds in which his income is \$74,999 or less.

That differential *no* is a degree quantifier follows from standard analyses of English comparatives, as we have shown; our proposal is that “determiner *no*” is (or at least can be) precisely the same degree quantifier, and that “split scope” readings like (1b) are simply the result of *no* taking scope above the modal. This analysis relies on the independent hypothesis that nominals contain a degree argument position, an assumption that is motivated by a long tradition of work on the compositional semantics of amount comparatives (Bresnan 1973, Cresswell 1977, Heim 1985, Hackl 2000, etc.) and more recent work on the semantics of modified and unmodified numerals (Nouwen 2010, Kennedy 2013). There are different implementations of this hypothesis; for simplicity we follow Grosu and Landman 1998 and assume that nominal degree arguments are introduced by an unpronounced occurrence of MANY, a type <d,et> expression that relates plural individuals to their cardinalities:

$$(8) \llbracket \text{MANY} \rrbracket = \lambda n \lambda x . \#(x) = n$$

Assuming existential closure over the individual argument, saturation of the degree argument with a numeral or comparative comparative derives existential truth conditions: “there is an x of cardinality n such that...” Saturation of this argument with the negative degree quantifier *no* results truth conditions that are equivalent to what we get on the standard treatment of *no* as a (type <et,<et,t>>) quantificational determiner. Under this view, a simple example like (9) has the LF in (9a), and the truth conditions in (9b).

(9) The company hired no employees.

a. $[\text{no}_1 [\text{the company hired } [t_1 \text{ MANY employees}]]]$

b. $\{n : \exists x[\text{hire}(\mathbf{c}, x) \ \& \ \#(x) = n \ \& \ \text{employee}(x)]\} = \emptyset$

The truth conditions in (9b) state that there is no degree (number) n such that the company hired a group of (at least) n employees; this will be so only when the company did not hire even one employee.

Returning to the case of “split scope” constructions like (1), we predict two possible LFs, differing in the relative position of degree quantifier *no* and the modal, just as we saw with (6): one in which *no* scopes below the modal (10a), and one in which it scopes above the modal (10b).

(10) a. $\forall w[\text{ACC}_@(w) \rightarrow \{n : \exists x[\text{fire}_w(\mathbf{c}, x) \ \& \ \#_w(x) = n \ \& \ \text{employee}_w(x)]\} = \emptyset]$

b. $\{n : \forall w[\text{ACC}_@(w) \rightarrow \exists x[\text{fire}_w(\mathbf{c}, x) \ \& \ \#_w(x) = n \ \& \ \text{employee}_w(x)]]\} = \emptyset$

The truth conditions in (10a) state that in every accessible world, there is no n such that the company fires a group of (at least) n employees, which forbids the firing of any employees. The truth conditions in (10b) state that there is no n such that for every world, there is a group of (at least) n employees fired in that world. This means that there are some worlds in which no employees are fired, since firing at least one employees in every world would mean that the set in (10b) contains at least the number 1, but it allows for the possibility of worlds in which some employees are fired. This is the split scope reading.

It is well-known that the interactions between English degree quantifiers and other operators are highly constrained (see e.g. Heim 2001). Wide scope for degree quantifiers is only possible with certain modals (e.g., *need to* and *required to*, but not *should* and *supposed to*) and is entirely impossible with quantificational DPs. It turns out that the exact same constraints govern the availability of split scope readings for English negative DPs (Potts 2000, von Stechow and Iatridou 2007, Iatridou and Sichel 2011): “*The company should fire no employees*,” for example, lacks a split reading. This parallelism follows directly from the analysis presented here, which is a working out of the one suggested by Heim: “split scope” is just the wide scope construal of the degree quantifier *no*; when such a construal is impossible the split scope reading disappears. This result represents an advantage of our analysis over previous approaches, which either do not capture this parallelism at all, or else fail to account for its full generality.